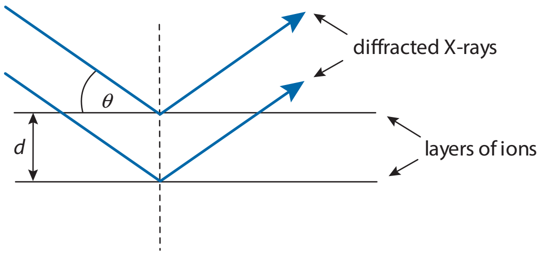
11

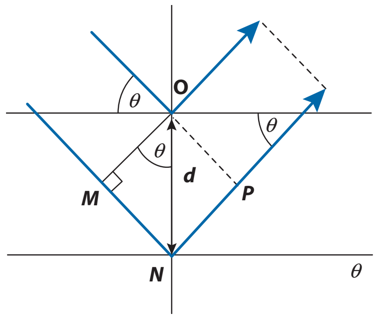
Trigonometry

Answers to additional problems

11.1



We first look closely at the inset below, which represents a magnification of the diagram above.



From the definition of a sin in eqn. (11.1) above, $\sin\theta$ = (opposite ÷ hypotenuse). The length of the hypotenuse is d and the length of the opposite is MN. Therefore,

$$\sin\theta = \frac{MN}{d}$$

The path length difference between the ray reflected at **O** and the ray reflected at N is $(MN + NP) = 2 \times MN$. Diffraction only occurs successfully when the length $2 \times MN$ comprises an integral number of wavelengths, call it $n \times \lambda$. We therefore substitute for the length MN, saying it is $n\lambda$

$$2 \times d \sin\theta = n\lambda$$

This yields the Bragg equation, $2d \sin\theta = n\lambda$.

11.2 This problem is asking us to rearrange an equation then correctly use a sine function.

We first rearrange the Bragg equation to make λ the subject,

$$\lambda = \frac{2d}{n}\sin\theta$$

Inserting values, $\lambda = \frac{2 \times 201\,\text{pm}}{1}\sin(34.68)$

so $\lambda = 402 \times 0.569$

and $\lambda = 229$ pm

- The units for d are the same as those of λ.

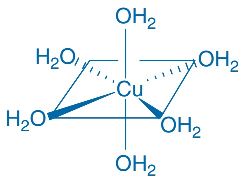
11.3 We first divide both sides by N, to make the sin function the subject

$$\frac{\psi}{N} = \sin\left(\frac{n\pi x}{L}\right)$$

then take the inverse of the sin $\sin^{-1}\left(\frac{\psi}{N}\right) = \frac{n\pi x}{L}$

Finally, we cross multiply $x = \frac{L}{n\pi}\sin^{-1}\left(\frac{\psi}{N}\right)$

11.4



VI

From the Pythagoras' theorem, $x^2 = (127\text{ pm})^2 + (127\text{ pm})^2 = 2 \times (127\text{ pm})^2$

$x^2 = 32\,258\text{ pm}^2$

so $x = \sqrt{32\,258\text{ pm}^2}$

and $x = 180$ pm

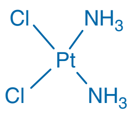
Distance between oxygen atoms = 180 pm.

11.5 We start with the cosine rule which says, $c^2 = a^2 + b^2 - 2ab\cos C$
When the triangle has a right angle, the angle opposite the hypotenuse θ is 90°; and cos 90° = 0. Therefore, the last part of the equation $2ab\cos C = 0$.
Accordingly, for a right-angled triangle, the cosine rule simplifies to become,

$$c^2 = a^2 + b^2 - 0$$

we've shown how the cosine rule becomes the Pythagoras' theorem $c^2 = a^2 + b^2$.

11.6



XI

Because each angle is 90°, we can use Pythagoras' theorem,

$x^2 = d^2_{(\text{Cl—Pt})} + d^2_{(\text{Pt—NH3})}$

$x^2 = (170\text{ pm})^2 + (162\text{ pm})^2$

so $x^2 = 28\,900\text{ pm}^2 + 26\,244\text{ pm}^2$

so $x^2 = 55\,144\text{ pm}^2$

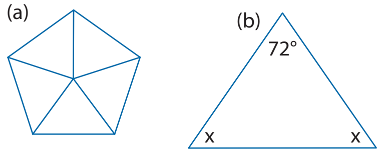
and $x = 235$ pm.

11.7

VII

Strategy

1. We consider the pentagon to comprise five regular triangles (see Fig. (a)).
2. We compute the angle at the centre using Fig. (b).
3. We compute the outer angles.

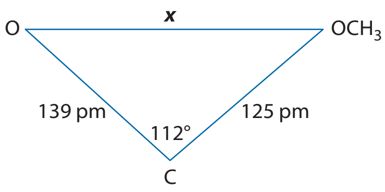


Solution

1. Because the polygon has five vertices, the internal angle will be one fifth of a full circle $= 360° \div 5 = 72°$
   A **polygon** is a plain shape with several straight sides. We usually only use this name for shapes in which each side has the same length.
2. The sum of the internal angles in a triangle is 180°. Looking at Fig. (b), we see how the other two angles (other than the 72°) will be the same. Accordingly, because all three of the internal angles of the triangle add up to 180°, $(2x + 72°) = 180°$, so $x = ½ (180 - 72)°$, so $x = 54°$.
3. The angle x, is *half* the C–C–C angle, so C–C–C = 108°.

To conclude, in degrees = 108° and in radians $= 2 \times \pi \times \left(\frac{108°}{360°}\right) = 0.6\pi = 1.885$ rad

11.8 We start by drawing the triangle,



From the cosine rule, eqn. (11.12), $c^2 = a^2 + b^2 - 2ab \cos C$

$$x^2 = (139\text{ pm})^2 + (125\text{ pm})^2 - (2 \times 139\text{ pm} \times 125\text{ pm}) \cos 112°$$

$$x^2 = 19\,321\text{ pm}^2 + 15\,625\text{ pm}^2 - (34\,750\text{ pm}^2) \times -0.374$$

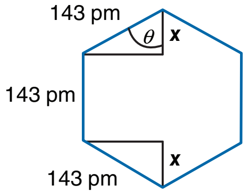
$$x^2 = 19\,321\text{ pm}^2 + 15\,625\text{ pm}^2 + 13\,018\text{ pm}^2$$

$$x^2 = 47\,964\text{ pm}^2$$

so $\quad x = \sqrt{47\,964\text{ pm}^2} = 219\text{pm}$

which is quite long, explaining why the interaction is weak.

11.9 We will redraw structure (**IX**) slightly Before we start the question,



The vertical distance from atom 1 to atom 4 is 143 pm, plus twice the distance x.
From a calculation similar to that in Additional Problem 11.7, angle $\theta = 60°$ so, using eqn. (11.2)

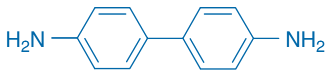
$$x = 143\text{ pm} \times \cos 60°$$

$$x = 143\text{ pm} \times 0.5$$

and $x = 71.5$ pm.

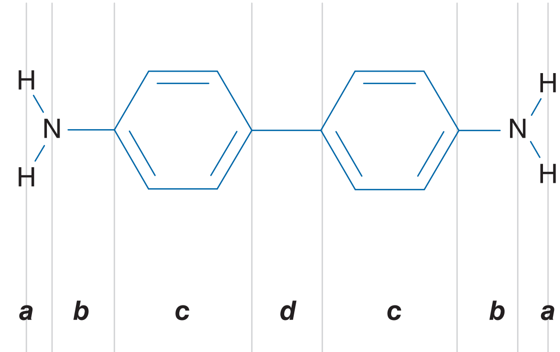
Therefore, the distance from atom 1 to atom 4 = (143 pm + 2 × 71.5 pm) = 286 pm.

11.10



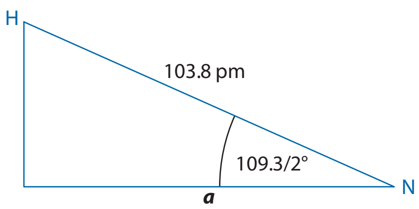
X

It's probably easiest if we start by redrawing (**X**) with all the distances drawn on it. In effect, we subdivide (**X**) into manageable lengths,



$\boldsymbol{a}$ = 60.1 pm,	as below
$\boldsymbol{b}$ = 139.0 pm	(from the question)
$\boldsymbol{c}$ = 286 pm	(Additional Problem 11.9)
$\boldsymbol{d}$ = 153 pm	(from the question)

We calculate length a using cosines, and the diagram below,



$$a = 103.8\text{ pm} \times \cos\left(\frac{109.3°}{2}\right)$$

$$\text{so } a = 103.8\text{ pm} \times 0.579$$

$$\text{so } a = 60.1\text{ pm}$$

The length of the molecule = $2a + 2b + 2c + d$ = 1123.2 pm.