Matrices I

23

Answers to additional problems

23.1 $$\begin{pmatrix} \alpha & \beta & 0 & 0 & \beta \\ \beta & \alpha & \beta & 0 & 0 \\ 0 & \beta & \alpha & \beta & 0 \\ 0 & 0 & \beta & \alpha & \beta \\ \beta & 0 & 0 & \beta & \alpha \end{pmatrix}$$

23.2 $$\begin{pmatrix} 0-0 & 0-3 & 5-(-3) \\ 2-3 & 4-2 & -9-1 \\ 1-2 & 4-(-2) & 6-0 \\ 3-6 & 3-7 & 1-2 \end{pmatrix} = \begin{pmatrix} 0 & -3 & 8 \\ -1 & 2 & -10 \\ -1 & 6 & 6 \\ -3 & -4 & -1 \end{pmatrix}$$

23.3 $$\begin{pmatrix} 2 & -3 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

Using eqn. (23.3), the determinant = (2 × 5) – (1 × –3) = 13. We can write an inverse matrix because this value is not zero.

Equation (23.11) gives the inverse of the square matrix as, $\frac{1}{13}\begin{pmatrix} 5 & 3 \\ -1 & 2 \end{pmatrix}$

$$\frac{1}{13}\begin{pmatrix} 5 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 9 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{13}\begin{pmatrix} (5\times 5)+(9\times 3) \\ (5\times -1)+(9\times 2) \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 52 \\ 13 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Therefore, $x = 4$, and $y = 1$.

23.4 Simple matrix multiplication yields the product, $\mathbf{B\,C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

For example, the first element a_{11} is (–15 × 1) + (2 × 2) + (–3 × –4) = –15 + 4 + 12 = 1. Since this 3 × 3 matrix is the identity matrix, **I** is proof positive that **B** and **C** are inverses. Similarly, **C B** = **I**.

23.5 $$\begin{vmatrix} a & 5b \\ 3ab^2 & 4a \end{vmatrix} = a \times 4a - 3ab^2 \times 5b = 4a^2 - 15ab^3 = a(4a - 15b^3)$$

23.6 $$\det \mathbf{A} = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 2 & 0 \\ 0 & 4 & 3 \end{vmatrix} = 1\begin{vmatrix} 2 & 0 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + 5\begin{vmatrix} 2 & 2 \\ 0 & 4 \end{vmatrix}$$

so **det A** = 1 × {(2×3) – (0×4)} – 2 × {(2×3) – (0×0)} + 5{(2×4) – (2×0)}
and **det A** = 6 – 12 + 40 = 34.

23.7
$$\begin{pmatrix} 1 & 9 & 6 & 2 & 2 & 0 \\ 6 & 2 & 3 & -1 & 8 & 1 \\ 1 & 0 & 6 & 31 & 13 & 1 \end{pmatrix} + \begin{pmatrix} -4 & 1 & 0 & 3 & -2 & 2 \\ 2 & 1 & 0 & 0 & 2 & 3 \\ 0 & 7 & 7 & 2 & 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1-4 & 9+1 & 6+0 & 2+3 & 2-2 & 0+2 \\ 6+2 & 2+1 & 3+0 & -1+0 & 8+2 & 1+3 \\ 1+0 & 0+7 & 6+7 & 31+2 & 13+1 & 1+0 \end{pmatrix}$$
$$= \begin{pmatrix} -3 & 10 & 6 & 5 & 0 & 2 \\ 8 & 3 & 3 & -1 & 10 & 4 \\ 1 & 7 & 13 & 33 & 14 & 1 \end{pmatrix}$$

23.8
$$3.2a\begin{pmatrix} a & 2 \\ a^2 & 3e \end{pmatrix} = \begin{pmatrix} 3.2a \times a & 3.2a \times 2 \\ 3.2a \times a^2 & 3.2a \times 3e \end{pmatrix} = \begin{pmatrix} 3.2a^2 & 6.4a \\ 3.2a^3 & 9.6ae \end{pmatrix}$$

23.9

	1	2	3	4	5	6	7	8	9	10
1	α	β	0	0	0	β	0	0	0	0
2	β	α	β	0	0	0	0	0	0	0
3	0	β	α	β	0	0	0	0	0	0
4	0	0	β	α	β	0	β	0	0	0
5	0	0	0	β	α	β	0	0	0	β
6	β	0	0	0	β	α	0	0	0	0
7	0	0	0	β	0	0	α	β	0	0
8	0	0	0	0	0	0	β	α	β	0
9	0	0	0	0	0	0	0	β	α	β
10	0	0	0	0	β	0	0	0	β	α

23.10

	1	2	3	4	5
1	1.5α	1.75β	0	0	1.75β
2	1.75β	α	β	0	0
3	0	β	α	β	0
4	0	0	β	α	β
5	1.75β	0	0	β	α